N.B. (i) All questions are compulsory.

(ii) Attempt ANY TWO -sub questions out of FOUR in Q.1, Q.2 and Q.3..

(iii) Attempt ANY THREE sub questions out of FOUR in Q.4.

(iv) Figures to the right indicate marks.

Q.1. (a) Define for a continuous random variable ,its   10
(i) probability density function
(ii) Cumulative distribution function.
State one property of each of them.

(b) For a continuous random variable , p.d.f. is   10

$$f(x) = k\,x \qquad\qquad 0 < x < 2$$
$$= 0 \qquad\qquad\qquad \text{otherwise}$$

Find k , mean and standard deviation of X.

(c) Determine cumulative distribution function F(x) for the following   10
probability density function.

$$f(x) = k\,x \qquad\qquad 0 \le x \le \tfrac{1}{2}$$
$$= 6\,(1-x) \qquad\qquad \tfrac{1}{2} \le x \le 1$$
$$= 0 \qquad\qquad\qquad \text{otherwise}$$

Where k is constant to be suitably chosen.

(d) Suppose the number of minutes a typist spends during an eight hour day for   10
non-productive activities has the p.d.f.

$$f(x) = \frac{k}{x3} \qquad\qquad x \ge 30$$
$$= 0 \qquad\qquad\qquad \text{otherwise}$$

Find k. Also determine the probability that on a certain day she wastes at
least 40 minutes of her working period.

Q.2. (a) State the probability density function of an exponential variate X with   10
parameter θ. Obtain mean of X. Also, state the variance of X.

(b) The demand of cakes (in kg.) at a bakery shows Rectangular distribution in   10
(1000,1500). Find the probability that on a certain day the demand would be
(i) at least 1200 kg.   (ii) between 1100 kg. and 1450 kg.

(c) Number of patients visiting a doctor's clinic follows Normal distribution   10
with mean 50 and standard deviation 10. How many days of a month (26
working days) does doctor expect (i) at least 60 patients   (ii) 35 to 55
patients ?

(d) A sales manager believes that 60% of the consumers prefer his product over   10
his competitors. Under the assumption of Normal distribution, what is the
probability of obtaining fewer than 54% who prefer his product in a random
sample of 100 consumers?

q

Q.3. (a) Explain the following terms with suitable examples: 10
  (i) Hypothesis
  (ii) Null Hypothesis
  (iii) Alternative Hypothesis
  (iv) Critical region
  (v) Type I Error and Type II Error

(b) A machine is set to produce metal plates of mean thickness 1.5 cm. with a   10
standard deviation of 0.2 cm. Find the probability that a sample of 400
plates drawn from the plates produced by the machine has mean thickness
within $\pm$ 0.01 cm. from 1.5 cm

(c) An item is packed in lots of 100 each. Let m denote the mean number of   10
defectives in a packet of 100. To test $H_0 : m = 2$ against $H_1 : m = 3$, it is
decided to select one packet and inspect the items in it. If it contains four or
more defectives, it is proposed to reject H_0. Find the level of significance
for the test. Also find Type II error.

(d) A pharmaceutical firm maintains that the mean time for a drug to show   10
effect is 24 minutes. In a sample of 400 trials, the mean time is 26 minutes
with a standard deviation of 4 minutes. Test the hypothesis that the mean
time is 24 minutes against the alternative that it is not equal to 24 minutes.
Use 5% level of significance.

Q.4. (a) X is a continuous random variable with p.d.f.   5

$$f(x) = \frac{k}{x2} \qquad x \geq 100$$
$$= 0 \qquad x < 100$$

Find k and the prob.$(x < 150)$

(b) If X follows Rectangular distribution in (a,b), write down p.d.f. of X. Hence   5
obtain expressions for mean and state variance of X.

(c) The income distribution of a group of 10000 persons was found to be   5
Normal with mean ₹ 750/- p.m. and standard deviation ₹ 50/- p.m. What
percentage of this group had income- exceeding ₹ 668/- (Given : Area
under standard normal curve between ordinates at $\pm$ 1.64 is 0.8990)

(d) A box contains 10 marbles out of which K are white and the rest are blue.   5
We want to test whether K = 5 or K = 4. If two marbles taken at random,
with replacement turn out to be blue, we accept K to be 4.
With reference to the above problem explain :
  (i) Null hypothesis and alternative hypothesis   (ii) Critical region
  (iii) Type I and Type II errors   (iv) Level of significance.

****